AP Calculus BC Math Curriculum

Egg Harbor Township School District

State Board Adoption Date of Standards: 5/2016

Unit Overview (Standards Coverage)				
Unit	**Standards**	**Unit Focus**	**Standards for Mathematical Practice**	**Open Educational Resources**
Unit 1 Limits and Continuity 15 days	• Limits • Continuity • Intermediate Value Theorem • Extreme Value Theorem • Asymptotes	How does knowing the value of a limit, or that a limit does not exist, help you to make sense of interesting features of functions and their graphs?	MP.1 Determine expressions and values using mathematical procedures and rules. MP.2 Translate mathematical information from a single representation or across multiple representations. MP.3 Justify reasoning and solutions. MP.4 Use correct notation, language, and mathematical conventions to communicate results or solutions.	AP Central Khan Academy Delta Math Wolfram Mathworld
Unit 2 Differentiation 38 days	• Derivative Rules • Using derivatives to determine properties of graphs • Rates of change • Mean Value Theorem • L'Hospital's Rule • Optimization • Connecting Position, Velocity and Acceleration	How do derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals?		AP Central Khan Academy Delta Math Wolfram Mathworld
Unit 3 Integration 41 days	• Integration Rules • Fundamental Theorems of Calculus • Area • Volume • Differential Equations • Advanced Integration Techniques • Euler's Method • Arc Length • Connecting Position, Velocity and Acceleration	How is integrating to find areas related to differentiating to find slopes?		AP Central Khan Academy Delta Math Wolfram Mathworld
Unit 4 Parametric Equations, Polar Coordinates, and Vector-Valued Functions	• Differentiation and integration of parametric equations • Differentiation and integration of vector-valued equations • Defining polar coordinates and differentiating in polar form.	How can we model motion not constrained to a linear path?		AP Central Khan Academy Delta Math Wolfram Mathworld

12 days	• Find the area of a region bounded by polar curves			
Unit 5 Infinite Sequences and Series 14 days	• Define convergent and divergent infinite series • Apply tests for convergence • Determine absolute or conditional convergence • Determine error bounds • Represent a function with a Taylor or Maclaurin Series • Represent a function as a Power Series	How can the sum of infinitely many discrete terms be a finite value or represent a continuous function?		AP Central Khan Academy Delta Math Wolfram Mathworld

This document outlines in detail the answers to the following four questions:

1. **What do we want our students to know?**
2. **How do we know if they learned it?**
3. **What do we do if they did not learn it?**
4. **What do we do when they did learn it?**

Unit 1 AP CALCULUS BC		
Content & Practice Standards	**Interdisciplinary Standards**	**Critical Knowledge & Skills**
• **Limits** • **Continuity** • **Intermediate Value Theorem** • **Extreme Value Theorem** • **Asymptotes**	• **WHST.11-12.10** • **Technology: 8.1A, C, E, F** • **21st Century Themes/Careers: CRP2, 4, 6, 8, 11**	Finding Limits to determine and evaluate Continuity. Students will utilize the Intermediate Value Theorem to solve problems. Utilizing and identifying asymptotes.

Unit 1 AP CALCULUS BC

Stage 1 – Desired Results

UNIT SUMMARY	CORE AND SUPPLEMENTAL MATERIALS/RESOURCES
During this unit students will review with the relationship between algebra/geometry and the development of Calculus. Evaluating limits both analytically and graphically is a major area of the unit and will be emphasized. Students will use the TI-89 calculator to help develop the intuitive feel of limits and graph behavior. From this unit students will have a complete understanding of limits and how they are used.	• Calculus of a Single Variable, 9th Edition, Larson, Hostetler, Edwards • AP Central • Graphing Calculator

UNDERSTANDINGS

Students will understand that the concept of a limit is one of the foundations of calculus.

Students will know…	Students will be able to…
• The connection of Precalculus to Calculus through limits • Continuous and Discontinuous functions and • how they relate to limits • The definition of limit • When limits fail to exist • How to use limit properties to evaluate limits • How the squeeze theorem is derived and its application to trigonometric limits • How continuity and limits are related • How continuity and one-sided limits are related • To find limits at infinity	• Explain why the concept of limit was important in solving the tangent line and area problems • Explain and write in mathematical terms the definition of limit. • Show geometrically when a limit does not exist and give algebraic examples of those functions. • State the relationship between continuity and limits vs discontinuity and limits • Use graphs, numerical tables, and algebra methods to find limits • Create a strategy which includes limit properties and methods to evaluate various limits. • Explain the IVT and EVT theorems verbally and geometrically • State how one-side limits help evaluate functions such as piecewise functions, step functions, and rational functions

Stage 2 – Assessment Evidence

Performance Tasks/Use of Technology	Formative
● Card Sorts ● Khan Academy ● Delta Math ● College Board	● Observation ● Homework ● Class Participation ● Whiteboards/communicators ● Think-Pair-Share ● Do-Now ● Notebook Checks ● Exit Tickets ● Classroom Games ● Card Sorts and Matching Activities ● Self-assessment **Summative** ● Chapter/Unit Quizzes ● Chapter/Unit Tests ● Unit Projects

Stage 3 – Learning Plan

Limits introduce the subtle distinction between evaluating a function at a point and considering what value the function is approaching, if any, as x approaches a point. This distinction allows us to extend understanding of asymptotes and holes in graphs with formal definitions of continuity. We will begin the year with a review of Limits covering the following:

- Introducing Calculus: Can Change Occur at an Instant?
- Defining Limits and Using Limit Notation
- Estimating Limit Values from Graphs and Tables
- Determining Limits Algebraically
- Selecting Procedures for Determining Limits
- Determining Limits Using the Squeeze Theorem
- Connecting Multiple Representations of Limits
- Exploring Types of Discontinuities, including at a point and over an interval
- Connecting Limits and Horizontal and Vertical Asymptotes
- Working with the Intermediate Value Theorem

Planned Differentiation & Interventions for Tiers I, II, III, ELL, SPED, and Gift & Talented Students

• Rethink and revise. Dig deeper into ideas at issue (through the faces of understanding). Revise, rehearse, and refine, as needed. Guide students in self-assessment and self-adjustment, based on feedback from inquiry, results, and discussion.

• Evaluate understandings. Reveal what has been understood through final performances and products. Involve students in a final self-assessment to identify remaining questions, set future goals, and point toward new units and lessons.

•Tailor (personalize) the work to ensure maximum interest and achievement. Differentiate the approaches used and provide sufficient options and variety (without compromising goals) to make it most likely that all students will be engaged and effective.

Gifted & Talented:

- "Differentiating the Lesson" in Big Ideas online resources for all sections
- "Additional Topics" in Big Ideas online resources to extend and enhance instruction
- Big Ideas Game Closet
- Big Ideas Differentiated Instruction options
- Big Ideas Mini-Assessments
- Design Challenges
- Student Choice/Driven Activities
- Group Projects
- MobyMax
- LinkIt!
- Rocket Math
- Intervention Central
- Do to Learn
- Differentiation Strategies for Math
- Discovery Education Math
- Everyday Mathematics
- Homework Spot
- Math Fact Fluency

Tier I:

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- Big Ideas MATH Pyramid of Tiered Interventions for additional resources
- Record and Practice Journal
- Differentiated Instruction options
- Fair Game Review
- Vocabulary Support Glossary resources
- Mini-Assessments
- Game Closet
- Lesson Tutorials
- Flash Cards
  Extended Time
- Flexible Grouping
- Small Group Instruction
- Peer Buddies
- Math Tutoring Center (HS only)

- Math Lab/Tutorial
- MobyMax
- LinkIt!
- Rocket Math
- Intervention Central
- Do to Learn
- Learning Ally
- Differentiation Strategies for Math
- Discovery Education Math
- Everyday Mathematics
- Homework Spot
- Flash Card Math
- Math Fact Fluency

Tier II:

- Lesson Tutorials
- Basic Skills Handbook
- Skills Review Handbook
- Differentiated Instruction Big Ideas resources
- Game Closet
- Centers/Small Group Instruction
- Math Tutoring Center (HS only)
- Math Lab/Tutorial
- MobyMax
- LinkIt!
- Math Fact Fluency/Rocket Math

Tier III:

- Customized Learning Intervention Activities resources
- Intensive Intervention resource
- Systematic Assessments to focus on specific deficits

ELL:

- Big Ideas Math Student Editions are available online in Spanish
- Letters to Parents are available in the Resources by Chapter book to assist in guiding parents through each chapter and offer helpful suggestions they can use to demonstrate mathematical concepts for their child in daily activities. These letters are editable so teachers can customize them.
- Student Dynamic eBook Audio has the option to be read in English or Spanish
- Multi-Language Glossary for new Math vocabulary is available in 14 different languages.
- Audio version is available in English or Spanish.
- Game Closet can be accessed in English or Spanish, while also allowing for all students to play and understand these educational games.

- ELL Notes included in Teacher Edition to help teachers overcome obstacles.
- Record & Practice Journal available in Spanish.
- Student Journal available in Spanish.
- Chapter Reviews available in English and Spanish.
- Vocabulary Flash Cards
- Chunking Information
- Math Word Wall/Word Bank
- Multi-Sensory Instruction
- Use of Translation software
- Gradual Release Model
- TODOS: Mathematics for ALL - Excellence and Equity in Mathematics
- FABRIC - A Learning Paradigm for ELLs (NJDOE resource)

SPED:

- Menu Math (mostly for very low functioning students)
- Math Labs/Tutorial
- MobyMax
- LinkIt!
- IXL
- Learning Ally (audio version for textbooks and other published materials) – Also available for 504 students
- Apex Online Learning – Bridge students only
- Use of specialized equipment such as beeping balls, text to speech and speech to text software, special seats or desks
- Use of hands-on materials for problem solving
- Visual supports and Use of Manipulatives
- Extended time to complete tests and assignments
- Graphic Organizers/Study Guides
- Mnemonic tricks to improve memory
- Reducing workload
- Centers/Small Group Instruction
- Adjusting accountability for standards by focusing only on essential standards
- Use of IPads or laptops for students with motor issues that make writing difficult
- Use of tangible rewards (certificates, small toys, etc. per behavior plan)
- Use prompts and model directions
- Use task analysis to break down activities and lessons into each individual step needed to complete the task
- Use concrete examples to teach concepts
- Have student repeat/rephrase written directions
- Provide multi-sensory, hands-on materials for instruction
- Chunking Information
- Modify all fine motor tasks for example: (fat crayons, pencil grip, adaptive scissors)
- Functional or practical emphasis

504:

- Learning Ally (audio version for textbooks and other published materials)
- Extra help opportunities
- Reduce workload
- Partial credit
- Allow use of calculator, when appropriate
- Modified length and time frame of assignments
- Alternate assessments with extended time
- Provide guided notes and study guides as needed (use interactive notebook)
- Preferential Seating
- Extra Practice
- Directions repeated, clarified and reworded
- Breakdown task into manageable units
- Differentiated instruction
- Use of manipulatives

Unit 2 AP CALCULUS BC		
Content & Practice Standards	**Interdisciplinary Standards**	**Critical Knowledge & Skills**
• **Derivative Rules** • **Using derivatives to determine properties of graphs** • **Rates of change** • **Mean Value Theorem** • **L'Hospital's Rule** • **Optimization** • **Connecting Position, Velocity and Acceleration**	• **WHST.11-12.10** • **Technology: 8.1A, C, E, F** • **21st Century Themes/Careers: CRP2, 4, 6, 8, 11**	Using derivatives and their rules to determine properties of graphs. Other key topics that will be covered are Rates of Change, Mean Value Theorem, L'Hospital's Rule, Optimization as well as Connecting Position, Velocity and Acceleration.

Unit 2 AP CALCULUS BC	
Stage 1 – Desired Results	
UNIT SUMMARY	**CORE AND SUPPLEMENTAL MATERIALS/RESOURCES**
In this unit we prepare the students for applications in differential calculus by giving them a firm grasp of methods of differentiation. Emphasis is placed on what a derivative represents (slope of a tangent line to a point on a curve), and the graphical differences between f(x) and f'(x), the relationship between differentiability and continuity is also a major point of interest in this unit.	• Calculus of a Single Variable, 9th Edition, Larson, Hostetler, Edwards • AP Central • Graphing Calculator
UNDERSTANDINGS	
Students will understand that derivatives allow us to determine instantaneous rates of change.	
Students will know…	**Students will be able to…**
• Derivatives of a function • Differentiability of a function • Rules for differentiation • Trigonometric derivatives • Implicit differentiation • Derivatives of Inverse Functions • Derivatives of Exponential and Logarithmic Functions • Whether a function is expressed explicitly of implicitly. • How to find a derivative using implicit differentiation.	• Find the first derivative and higher order derivatives of any explicit or implicit function. • Recognize the structure of the function and understand what rules apply and in what order to perform the rules • Use the derivative to find a linear approximation. • Use the derivative to find rates of change for various application problems, including but not limited to physics and/or business • Distinguish between functions written in explicit form and implicit form. • Find the derivative of a function expressed implicitly.

• When to use the appropriate differentiation rules as they implicitly differentiate. • The various methods for finding extrema on both a closed interval and an open interval. • The relationship between position, velocity and acceleration. • The Mean value Theorem guarantees that functions increase/decrease depending on the value of the derivative. • The first and second derivatives indicate the behavior of the original function.	• Use the rules for differentiation when differentiating implicitly. • Set up and solve related rates problems. • Locate extrema on a closed interval. • Determine if Rolle's Theorem is applicable and if so, apply the theorem. • Determine if the Mean Value Theorem is applicable and if so find the values guaranteed by the theorem. • Know the relationship between the position function, the velocity function and the acceleration function. • Use the Mean Value Theorem to determine the point at which the derivative equals the average rate of change. • Use the first and second derivative tests to determine maxima, points of inflection, intervals of increase/decrease, and intervals of concavity.

Stage 2 – Assessment Evidence

Performance Tasks/Use of Technology • Card Sorts • Khan Academy • Delta Math • College Board	**Formative** • Observation • Homework • Class Participation • Whiteboards/communicators • Think-Pair-Share • Do-Now • Notebook Checks • Exit Tickets • Classroom Games • Card Sorts and Matching Activities • Self-assessment **Summative** • Chapter/Unit Quizzes • Chapter/Unit Tests • Unit Projects

Stage 3 – Learning Plan

The concept of a derivative will be explored through multiple representations. Basic derivative rules will be reviewed and expanded upon, including the following:

- Defining Average and Instantaneous Rates of Change at a Point
- Defining the Derivative of a Function Using Derivative Notation
- Estimating Derivatives of a Function at a Point

- Connecting Differentiability and Continuity
- Applying Derivative Rules such as Power, Constant, Constant Multiple, Sum, Difference, Quotient
- Derivatives of Polynomial; Trigonometric;and Transcendental Functions
- Higher-Order Derivatives
- Using Derivatives to solve problems: Related Rates, Optimization, Straight Line Motion
- Using Derivatives to sketch curves
- The Chain Rule
- Implicit Differentiation
- Differentiating Inverse Functions
- L'Hopital's Rule
- Using the Mean Value Theorem and Extreme Value Theorem

Planned Differentiation & Interventions for Tiers I, II, III, ELL, SPED, and Gift & Talented Students

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- Visual supports and Use of Manipulatives

- Extended time to complete tests and assignments
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- Mnemonic tricks to improve memory
- Reducing workload
- Centers/Small Group Instruction
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- Preferential Seating
- Extra Practice
- Directions repeated, clarified and reworded
- Breakdown task into manageable units
- Differentiated instruction
- Use of manipulatives

Unit 3 AP CALCULUS BC		
Content & Practice Standards	**Interdisciplinary Standards**	**Critical Knowledge & Skills**
• **Integration Rules** • **Fundamental Theorems of Calculus** • **Area** • **Volume** • **Differential Equations** • **Advanced Integration Techniques** • **Euler's Method** • **Arc Length** • **Connecting Position, Velocity and Acceleration**	• **WHST.11-12.10** • **Technology: 8.1A, C, E, F** • **21st Century Themes/Careers: CRP2, 4, 6, 8, 11**	**Students will find Area and Volume, Arc Length and Differential Equations. Methods that will be used include Integration Rules, Fundamental Theorems of Calculus, Advanced Integration Techniques, Euler's Method. Students will also connect Position, Velocity and Acceleration.**

Unit 3 AP CALCULUS BC	
Stage 1 – Desired Results	
UNIT SUMMARY	**CORE AND SUPPLEMENTAL MATERIALS/RESOURCES**
This unit will supply the students with the capability of integrating a variety of function types. It is necessary for them to integrate by hand as well as with a calculator for the exam.	• Calculus of a Single Variable, 9th Edition, Larson, Hostetler, Edwards • AP Central • Graphing Calculator

UNDERSTANDINGS
Students will understand that there is a relationship between differentiation and integration and that the anti-derivative of a function and that the accumulation of change is related to the area under a curve.

Students will know…	**Students will be able to…**
• The integral gives the area under the curve. • The rules for anti-differentiation. • The area under a curve is the amount of change of a function over time. • Slope fields represent the numeric derivative of a function at any point • There are various techniques needed for integration. • How to find displacement and/or total distance from a given velocity.	• Evaluate an integral, definite or indefinite. • Apply the Fundamental Theorem of Calculus • Use integration by substitution and integration by parts as needed to evaluate integrals • Sketch a slope field • Match a differential equation with its slope field. • Solve a separable differential equation using substitution and partial fraction decomposition. • Use Euler's Method to solve an initial value differential value at a given point.

● How to find boundaries for integrating between intersecting curves. ● When to use horizontal or vertical rectangles for finding area based upon given conditions. ● You can compute the areas and volumes manually and through the use of available technology.	● Find displacement an object at a given moment. ● Find the total distance traveled over a given interval of time. ● Use algebraic methods or available technology to find point(s) of intersection that will be the boundaries of bounded regions. ● Use algebraic methods or available technology to find areas and volumes. ● Use dimensional analysis to determine what quantities and units of measure are calculated. For example – the calculated area under velocity curve (ft/sec) for a given time interval (in sec.) would result in how far (ft.) the object travels.

Stage 2 – Assessment Evidence

Performance Tasks/Use of Technology

- Card Sorts
- Khan Academy
- Delta Math
- College Board

Formative

- Observation
- Homework
- Class Participation
- Whiteboards/communicators
- Think-Pair-Share
- Do-Now
- Notebook Checks
- Exit Tickets
- Classroom Games
- Card Sorts and Matching Activities
- Self-assessment

Summative

- Chapter/Unit Quizzes
- Chapter/Unit Tests
- Unit Projects

Stage 3 – Learning Plan

This unit establishes the relationship between differentiation and integration using the following concepts:

- Accumulations of Change
- Approximating Areas with Riemann Sums
- Summation Notation and Definite Integral Notation
- The Fundamental Theorem of Calculus, Accumulation Functions and Definite Integrals
- Interpreting the Behavior of Accumulation Functions Involving Area
- Applying Properties of Definite Integrals

- Finding Antiderivatives and Indefinite Integrals
- Integration Using Substitution
- Integrating Functions Using Long Division and Completing the Square
- Integrating Using Integration by Parts
- Integrating Using Linear Partial Fractions
- Evaluating Improper Integrals
- Selecting Techniques for Antidifferentiation
- Finding the Average Value of a Function on an Interval
- Connecting Position, Velocity and Acceleration of Functions Using Integrals
- Finding Area Between Curves Expressed as Functions of x
- Volumes of Solids of Revolution
- Arc Length of a Smooth Planar Curve

Planned Differentiation & Interventions for Tiers I, II, III, ELL, SPED, and Gift & Talented Students

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Tier II:

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- Use of hands-on materials for problem solving
- Visual supports and Use of Manipulatives
- Extended time to complete tests and assignments
- Graphic Organizers/Study Guides
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- Breakdown task into manageable units
- Differentiated instruction
- Use of manipulatives

Unit 4 AP CALCULUS BC		
Content & Practice Standards	**Interdisciplinary Standards**	**Critical Knowledge & Skills**
• **Differentiation and integration of parametric equations** • **Differentiation and integration of vector-valued equations** • **Defining polar coordinates and differentiating in polar form.** • **Find the area of a region bounded by polar curves**	• **WHST.11-12.10** • **Technology: 8.1A, C, E, F** • **21st Century Themes/Careers: CRP2, 4, 6, 8, 11**	Differentiation and integration of parametric equations and vector-valued equations. Defining polar coordinates and differentiating in polar form. Finding the area of a region bounded by polar curves.

Unit 4 AP CALCULUS BC

Stage 1 – Desired Results

UNIT SUMMARY	CORE AND SUPPLEMENTAL MATERIALS/RESOURCES
In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas.	• Calculus of a Single Variable, 9th Edition, Larson, Hostetler, Edwards • AP Central • Graphing Calculator

UNDERSTANDINGS

Students will understand that motion can be modeled when not constrained to a linear path.

Students will know…	Students will be able to…
• Methods for calculating derivatives of real-valued functions can be extended to parametric functions. • The length of a parametrically defined curve can be calculated using a definite integral. • Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions. • Methods for calculating integrals of real-valued functions can be extended to vector-valued functions.	• Find the derivative and integral of a parametric equation • Convert between rectangular and parametric equations • Find the arc length of a parametric function • Find the derivative and integral of vector-valued functions • Solve motion problems using parametric equations and vector-valued functions • Convert between rectangular and polar coordinates • Find the area of a region bounded by a polar curve or curves

● Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along a curve in the plane defined using parametric or vector-valued functions. ● The definite integral of speed represents the particle's total distance traveled over the interval of time. ● Methods for calculating derivatives of real-valued functions can be extended to functions in polar coordinates. ● Methods for calculating integrals of real-valued functions can be extended to functions in polar coordinates. ● The concept of calculating areas in rectangular coordinates can be extended to polar coordinates. ● Areas of regions bounded by polar curves can be calculated with definite integrals.	

Stage 2 – Assessment Evidence

Performance Tasks/Use of Technology ● Card Sorts ● Khan Academy ● Delta Math ● College Board	**Formative** ● Observation ● Homework ● Class Participation ● Whiteboards/communicators ● Think-Pair-Share ● Do-Now ● Notebook Checks ● Exit Tickets ● Classroom Games ● Card Sorts and Matching Activities ● Self-assessment **Summative** ● Chapter/Unit Quizzes ● Chapter/Unit Tests ● Unit Projects

Stage 3 – Learning Plan

This unit will be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize. Concepts will include the following:

- Defining and Differentiating Parametric Equations

- Second Derivatives of Parametric Equations
- Finding Arc Lengths of Curves GIven by Parametric Equations
- Defining and Differentiating Vector-Valued Functions
- Integrating Vector-Valued Function
- Solving Motion Problems Using Parametric and Vector-Valued Functions
- Defining Polar Coordinates and Differentiating in Polar Form
- Finding the Area of a Polar Region or the Area Bounded by One or More Polar Curves

Planned Differentiation & Interventions for Tiers I, II, III, ELL, SPED, and Gift & Talented Students

• Rethink and revise. Dig deeper into ideas at issue (through the faces of understanding). Revise, rehearse, and refine, as needed. Guide students in self-assessment and self-adjustment, based on feedback from inquiry, results, and discussion.

• Evaluate understandings. Reveal what has been understood through final performances and products. Involve students in a final self-assessment to identify remaining questions, set future goals, and point toward new units and lessons.

•Tailor (personalize) the work to ensure maximum interest and achievement. Differentiate the approaches used and provide sufficient options and variety (without compromising goals) to make it most likely that all students will be engaged and effective.

Gifted & Talented:

- "Differentiating the Lesson" in Big Ideas online resources for all sections
- "Additional Topics" in Big Ideas online resources to extend and enhance instruction
- Big Ideas Game Closet
- Big Ideas Differentiated Instruction options
- Big Ideas Mini-Assessments
- Design Challenges
- Student Choice/Driven Activities
- Group Projects
- MobyMax
- LinkIt!
- Rocket Math
- Intervention Central
- Do to Learn
- Differentiation Strategies for Math
- Discovery Education Math
- Everyday Mathematics
- Homework Spot
- Math Fact Fluency

Tier I:

- "Differentiating the Lesson" in Big Ideas online resources for all sections
- Big Ideas MATH Pyramid of Tiered Interventions for additional resources
- Record and Practice Journal
- Differentiated Instruction options
- Fair Game Review
- Vocabulary Support Glossary resources
- Mini-Assessments
- Game Closet
- Lesson Tutorials
- Flash Cards
  Extended Time
- Flexible Grouping
- Small Group Instruction
- Peer Buddies
- Math Tutoring Center (HS only)
- Math Lab/Tutorial
- MobyMax
- LinkIt!
- Rocket Math
- Intervention Central
- Do to Learn
- Learning Ally
- Differentiation Strategies for Math
- Discovery Education Math
- Everyday Mathematics
- Homework Spot
- Flash Card Math
- Math Fact Fluency

Tier II:

- Lesson Tutorials
- Basic Skills Handbook
- Skills Review Handbook
- Differentiated Instruction Big Ideas resources
- Game Closet
- Centers/Small Group Instruction
- Math Tutoring Center (HS only)
- Math Lab/Tutorial
- MobyMax
- LinkIt!

- Math Fact Fluency/Rocket Math

Tier III:

- Customized Learning Intervention Activities resources
- Intensive Intervention resource
- Systematic Assessments to focus on specific deficits

ELL:

- Big Ideas Math Student Editions are available online in Spanish
- Letters to Parents are available in the Resources by Chapter book to assist in guiding parents through each chapter and offer helpful suggestions they can use to demonstrate mathematical concepts for their child in daily activities. These letters are editable so teachers can customize them.
- Student Dynamic eBook Audio has the option to be read in English or Spanish
- Multi-Language Glossary for new Math vocabulary is available in 14 different languages.
- Audio version is available in English or Spanish.
- Game Closet can be accessed in English or Spanish, while also allowing for all students to play and understand these educational games.
- ELL Notes included in Teacher Edition to help teachers overcome obstacles.
- Record & Practice Journal available in Spanish.
- Student Journal available in Spanish.
- Chapter Reviews available in English and Spanish.
- Vocabulary Flash Cards
- Chunking Information
- Math Word Wall/Word Bank
- Multi-Sensory Instruction
- Use of Translation software
- Gradual Release Model
- TODOS: Mathematics for ALL - Excellence and Equity in Mathematics
- FABRIC - A Learning Paradigm for ELLs (NJDOE resource)

SPED:

- Menu Math (mostly for very low functioning students)
- Math Labs/Tutorial
- MobyMax
- LinkIt!
- IXL
- Learning Ally (audio version for textbooks and other published materials) – Also available for 504 students
- Apex Online Learning – Bridge students only
- Use of specialized equipment such as beeping balls, text to speech and speech to text software, special seats or desks
- Use of hands-on materials for problem solving
- Visual supports and Use of Manipulatives

- Extended time to complete tests and assignments
- Graphic Organizers/Study Guides
- Mnemonic tricks to improve memory
- Reducing workload
- Centers/Small Group Instruction
- Adjusting accountability for standards by focusing only on essential standards
- Use of IPads or laptops for students with motor issues that make writing difficult
- Use of tangible rewards (certificates, small toys, etc. per behavior plan)
- Use prompts and model directions
- Use task analysis to break down activities and lessons into each individual step needed to complete the task
- Use concrete examples to teach concepts
- Have student repeat/rephrase written directions
- Provide multi-sensory, hands-on materials for instruction
- Chunking Information
- Modify all fine motor tasks for example: (fat crayons, pencil grip, adaptive scissors)
- Functional or practical emphasis

504:

- Learning Ally (audio version for textbooks and other published materials)
- Extra help opportunities
- Reduce workload
- Partial credit
- Allow use of calculator, when appropriate
- Modified length and time frame of assignments
- Alternate assessments with extended time
- Provide guided notes and study guides as needed (use interactive notebook)
- Preferential Seating
- Extra Practice
- Directions repeated, clarified and reworded
- Breakdown task into manageable units
- Differentiated instruction
- Use of manipulatives

Unit 5 AP CALCULUS BC		
Content & Practice Standards	**Interdisciplinary Standards**	**Critical Knowledge & Skills**
• **Define convergent and divergent infinite series** • **Apply tests for convergence** • **Determine absolute or conditional convergence** • **Determine error bounds** • **Represent a function with a Taylor or Maclaurin Series** • **Represent a function as a Power Series**	• **WHST.11-12.10** • **Technology: 8.1A, C, E, F** • **21st Century Themes/Careers: CRP2, 4, 6, 8, 11**	Defining convergent and divergent infinite series, applying tests for convergence, determining error bounds and determining absolute or conditional convergence. Representing a function with a Taylor or Maclaurin Series and representing a function as a Power Series.

Unit 5 AP CALCULUS BC

Stage 1 – Desired Results

UNIT SUMMARY	CORE AND SUPPLEMENTAL MATERIALS/RESOURCES
Students need to understand that a sum of infinitely many terms may converge to a finite value. They can develop intuition by exploring graphs, tables, and symbolic expressions for series that converge and diverge and for Taylor polynomials. Students should build connections to past learning, such as how evaluating improper integrals relates to the integral test or how using limiting cases of power series to represent continuous functions relates to differentiation and integration.	• Calculus of a Single Variable, 9th Edition, Larson, Hostetler, Edwards • AP Central • Graphing Calculator

UNDERSTANDINGS

A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.

Students will know…	Students will be able to…
• The nth partial sum is defined as the sum of the first n terms of a series. • An infinite series of numbers converges to a real number S (or has sum S), if and only if the limit of its sequence of partial sums exists and equals S. • A geometric series is a series with a constant ratio between successive terms. • The nth term test is a test for divergence of a series. • The integral test is a method to determine whether a series converges or diverges.	• Determine whether a series converges or diverges • Approximate the sum of a series • Represent a function at a point as a Taylor polynomial • Determine the error bound associated with a Taylor polynomial approximation • Determine the radius and interval of convergence for a power series • Represent a function as a Taylor or Maclaurin series • Represent a function as a power series

● In addition to geometric series, common series of numbers include the harmonic series, the alternating harmonic series, and p-series. ● The comparison test is a method to determine whether a series converges or diverges. ● The limit comparison test is a method to determine whether a series converges or diverges. ● The alternating series test is a method to determine whether an alternating series converges. ● The ratio test is a method to determine whether a series of numbers converges or diverges. ● A series may be absolutely convergent, conditionally convergent, or divergent. ● If a series converges absolutely, then it converges. ● If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value. ● If an alternating series converges by the alternating series test, then the alternating series error bound can be used to bound how far a partial sum is from the value of the infinite series. ● In many cases, as the degree of a Taylor polynomial increases, the nth degree polynomial will approach the original function over some interval. ● Taylor polynomials for a function f centered at x = a can be used to approximate function values of f near x = a. ● The Lagrange error bound can be used to determine a maximum interval for the error of a Taylor polynomial approximation to a function. ● In some situations, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the value of a function. ● If a power series converges, it either converges at a single point or has an interval of convergence. ● The ratio test can be used to determine the radius of convergence of a power series. ● The radius of convergence of a power series can be used to identify an open interval on which the series converges, but it is necessary to test both endpoints of the interval to determine the interval of convergence. ● If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval.	

- The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series.
- A Taylor polynomial for f (x) is a partial sum of the Taylor series for f (x).
- The Maclaurin series for sin x, cos x, and e^x provides the foundation for constructing the Maclaurin series for other functions.
- The Maclaurin series for 1/(1-x) is a geometric series.
- Using a known series, a power series for a given function can be derived using operations such as term-by-term differentiation or term-by-term integration, and by various methods (e.g., algebraic processes, substitutions, or using properties of geometric series).

Stage 2 – Assessment Evidence

Performance Tasks/Use of Technology

- Card Sorts
- Khan Academy
- Delta Math
- College Board

Formative

- Observation
- Homework
- Class Participation
- Whiteboards/communicators
- Think-Pair-Share
- Do-Now
- Notebook Checks
- Exit Tickets
- Classroom Games
- Card Sorts and Matching Activities
- Self-assessment

Summative

- Chapter/Unit Quizzes
- Chapter/Unit Tests
- Unit Projects

Stage 3 – Learning Plan

In this unit, students will build connections to past learning, such as how evaluating improper integrals relates to the integral test or how using limiting cases of power series to represent continuous functions relates to differentiation and integration through the following:

- Definitions of Convergent and Divergent Infinite Series
- Geometric Series
- nth Term Test for Divergence
- Integral Test for Convergence

- Harmonic Series and p-Series
- Comparison Tests for Convergence
- Alternating Series Test for Convergence
- Ratio Test for Convergence
- Absolute vs. Conditional Convergence
- Alternating Series Error Bound
- Taylor Polynomial Approximations of Functions
- Lagrange Error Bound
- Radius and Interval of Convergence of Power Series
- Taylor and Maclaurin Series for a Function
- Representing Functions as Power Series

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